CS 410/510: Advanced Programming

Lecture 7: Hamming, Closures, Laziness

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The Hamming Set:

$$
\begin{aligned}
\text{hamming} = \{\, 1 \,\}\; & \cup \{\, 2 * x \mid x \in \text{hamming} \,\} \\
& \cup \{\, 3 * x \mid x \in \text{hamming} \,\} \\
& \cup \{\, 5 * x \mid x \in \text{hamming} \,\}
\end{aligned}
$$

$$
\text{hamming} = \{\, 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18, 20, 24, \ldots \,\}
$$

The Hamming Sequence:

```
hamming = 1 :
            (merge [ 2 * x | x <- hamming ]
            (merge [ 3 * x | x <- hamming ]
                   [ 5 * x | x <- hamming ]))

Main> hamming
[ 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18,
  20, 24, ... ^C{Interrupted!}
Main>
```

The Hamming Sequence:

```
hamming = 1 :
            (merge (map (2*) hamming)
            (merge (map (3*) hamming)
                   (map (5*) hamming)))
```

```
Main> hamming
[ 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 16, 18,
  20, 24, … ^C{Interrupted!}
Main>
```

How does this work?

“Infinite” Lists in Haskell:

How do examples like the following work?

```
Main> [1..]
[1,2,3,4,5,6,7,8,9,10,11^C{Interrupted!}

Main> iterate (10*) 1
[1,10,100,1000,10000,100000,1000000^C{Interrupted!}

Main> fibs where fibs = 0 : 1 : [ x+y | (x,y) <- zip fibs (tail fibs) ]
[0,1,1,2,3,5,8,13,21,34,55,89,144,233, ^C{Interrupted!}

Main>
```

Closures, Delays, Thunks …

- Haskell Expressions are treated as:
  - Thunks
  - Closures
  - Delayed Computations
  - Suspensions
  - …
- Expressions are evaluated:
  - Lazily
  - On demand
  - By need
  - …

[1..]

The list [1..] is syntactic sugar for the expression enumFrom 1, where:

```
enumFrom n = n : enumFrom (n+1)
```

enumFrom	n

Code: instructions on how to produce the next element

Data: inputs that are needed to produce the next element

Closure/Thunk

[n..m]

The list [n..m] is syntactic sugar for the expression enumFromTo n m, where:

```
enumFromTo n m
    = if n<=m then n : enumFromTo (n+1) m
                  else []
```

enumFromTo	n, m

Code: instructions on how to produce the next element

Data: inputs that are needed to produce the next element

Closure/Thunk

sum [1..10]

```
sum xs = sum’ 0 xs
    where sum’ n [] = n
          sum’ n (x:xs) = sum’ (n+x) xs
```

```
sum [1..10]
= sum’ 0 [1..10]
= sum’ 1 [2..10]
= sum’ 3 [3..10]
= sum’ 6 [4..10]
= …
= sum’ 55 [11..10]
= 55
```

```
t :=0; n:=1; m:=10;
while (n<=m) {
   t  := t + n;
   n := n+1;
}
```

sum’ t [n..m]

Closures in Smalltalk:

- Blocks provide a similar mechanism:
  - [i := i + 1] describes a computation, but doesn’t run it (yet)
  - aBlock value forces
- Essential to make control structures work:
  - aBool ifTrue: […] ifFalse: […]
- A bigger example:
  - BlockClosure>>doWhileFalse: conditionBlock
  - |result|
  - [result := self value. conditionBlock value] whileFalse.
  - ^ result

[1..]

In Smalltalk:

- A class EnumFrom, instance variable head

- A class method: EnumFrom with: head

- Accessor methods:

```
EnumFrom>>> head
^ head

EnumFrom>>> tail
^ EnumFrom with: (head+1)
```

map (mult*)

In Smalltalk:

- A class MultiplyBy, instance variables mult, aList
- A method: aList multiplyBy: mult

  (Which class should be home to this code?)
- Accessor methods:

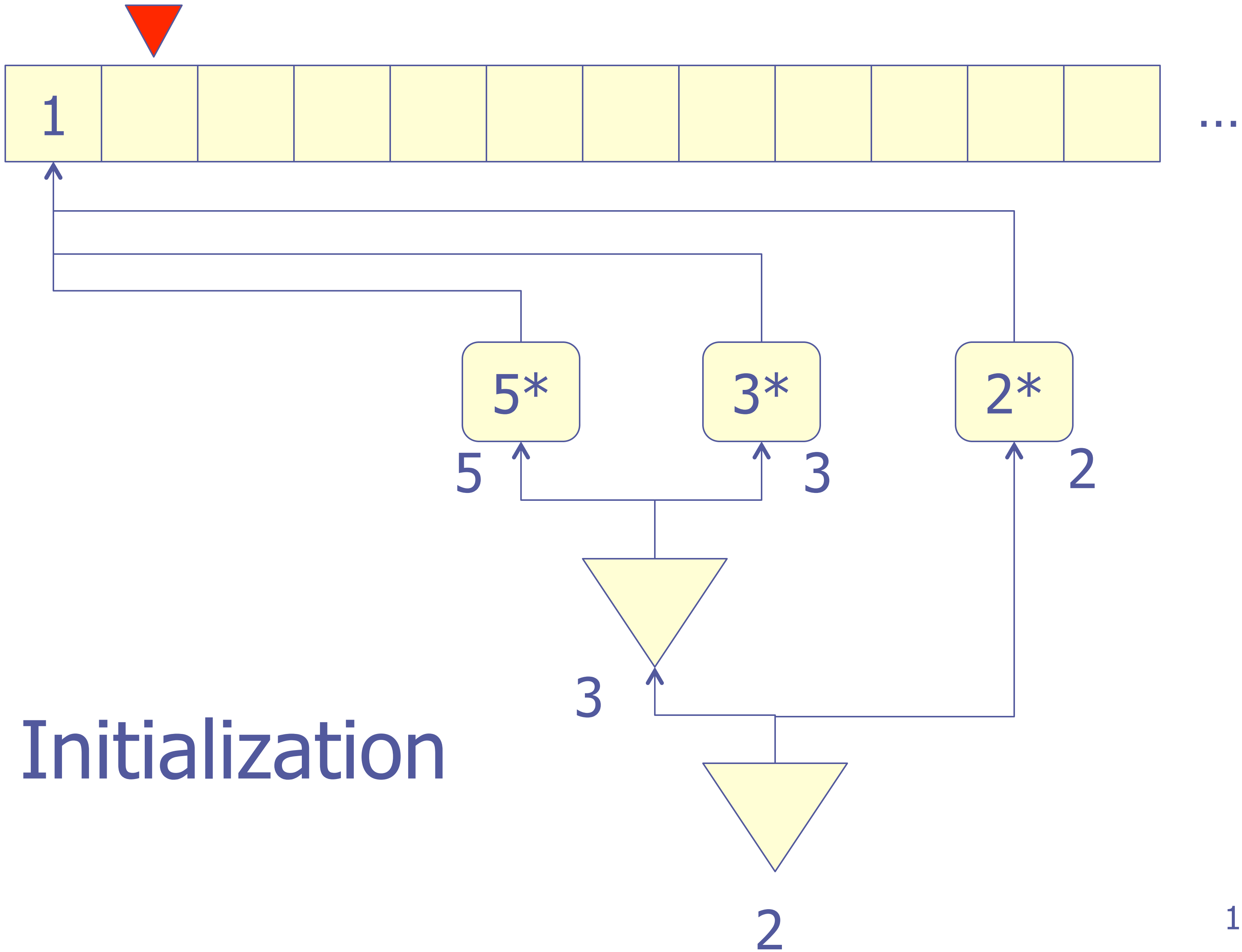
  EnumFrom>>> head

  ^ aList head * mult

  EnumFrom>>> tail

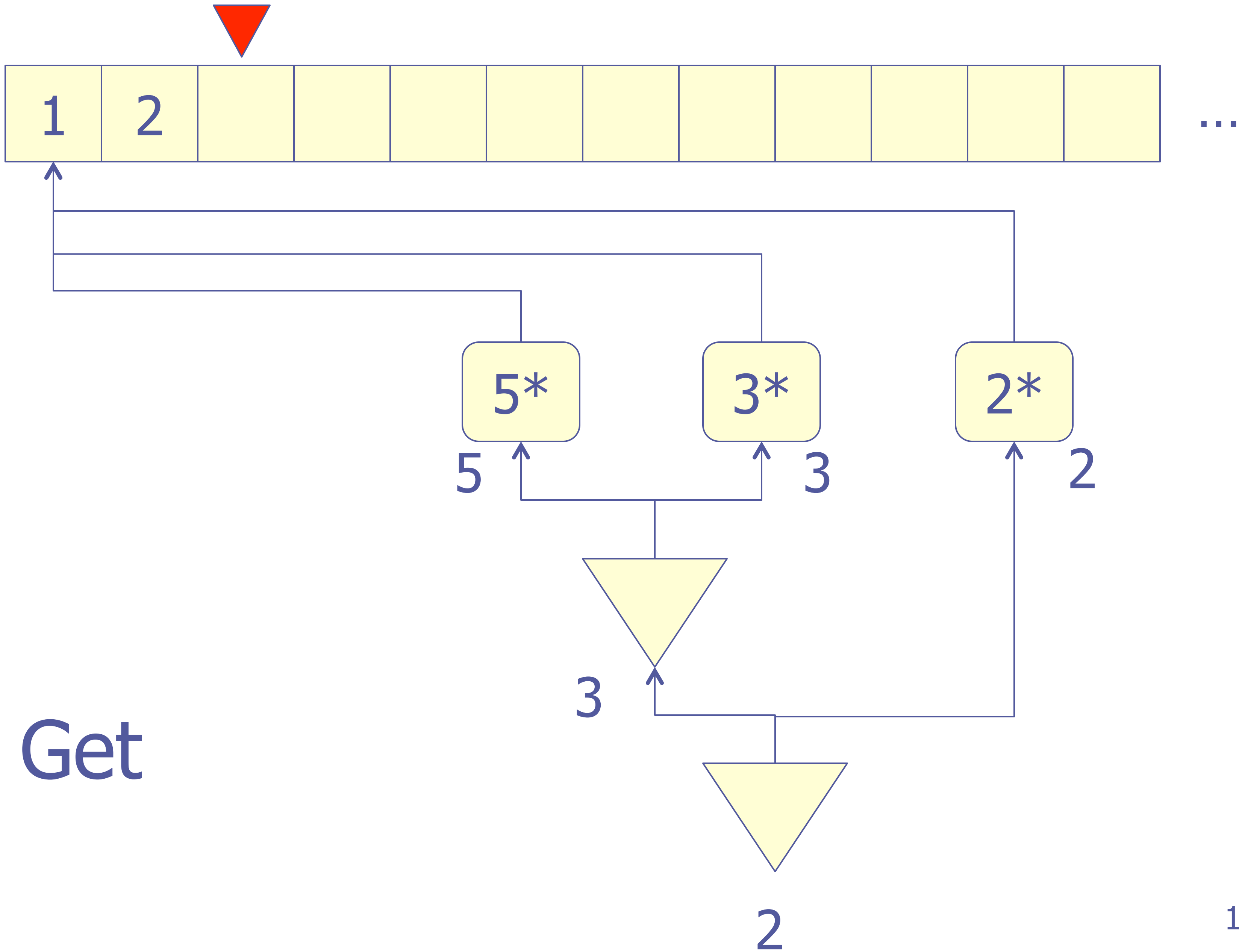
  ^ aList tail multiplyBy: mult

The Hamming Sequence:



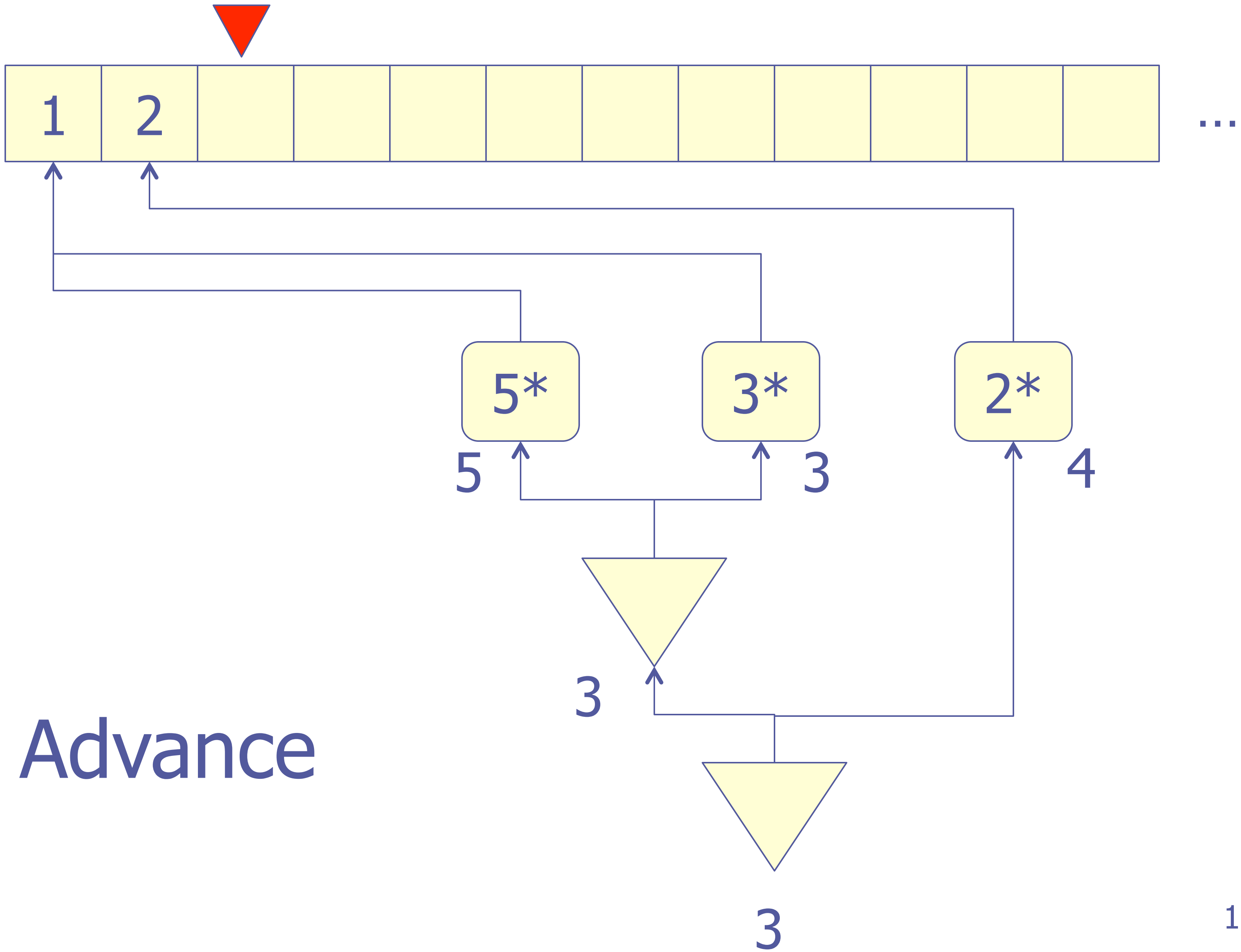
Initialization

The Hamming Sequence:



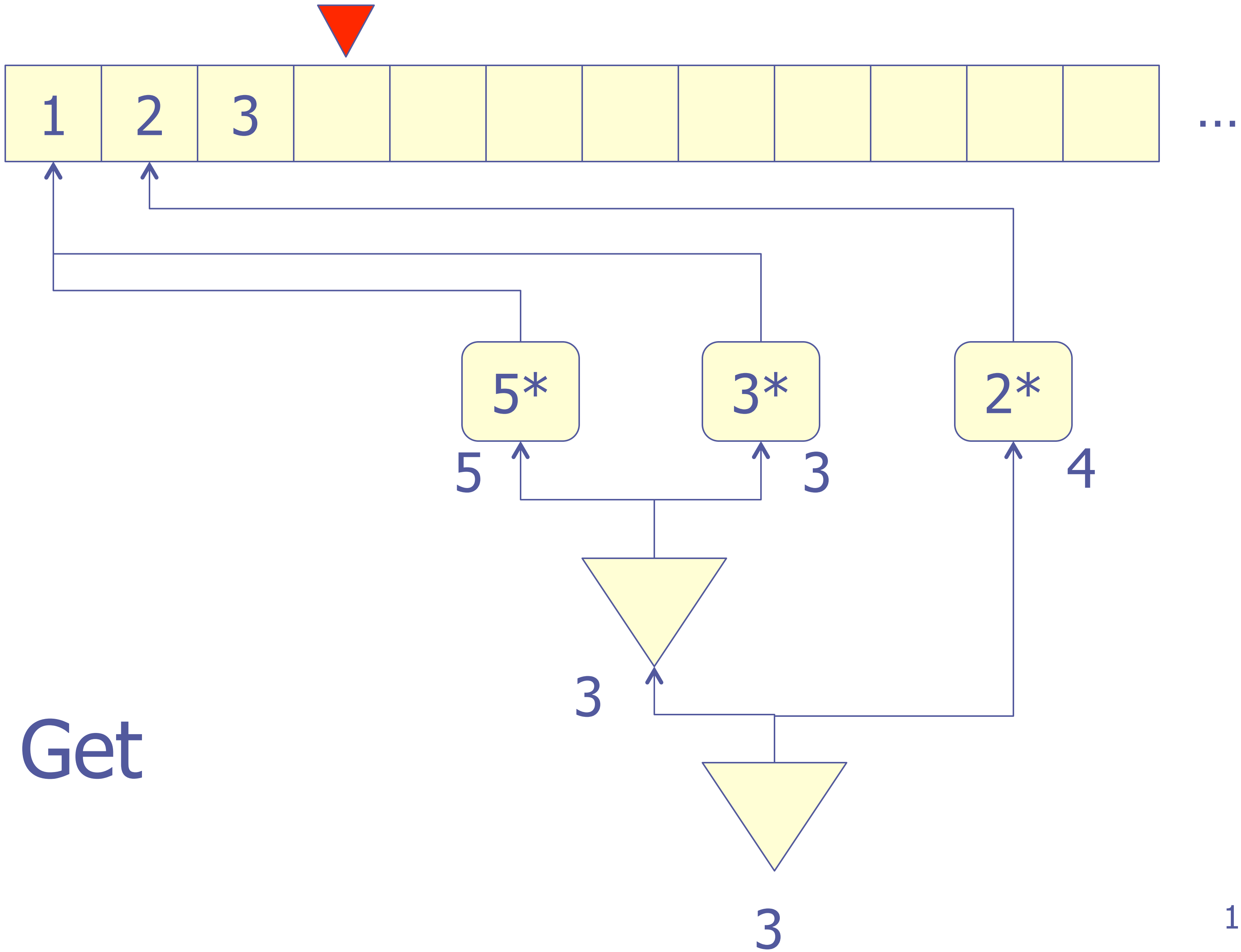
Get

The Hamming Sequence:



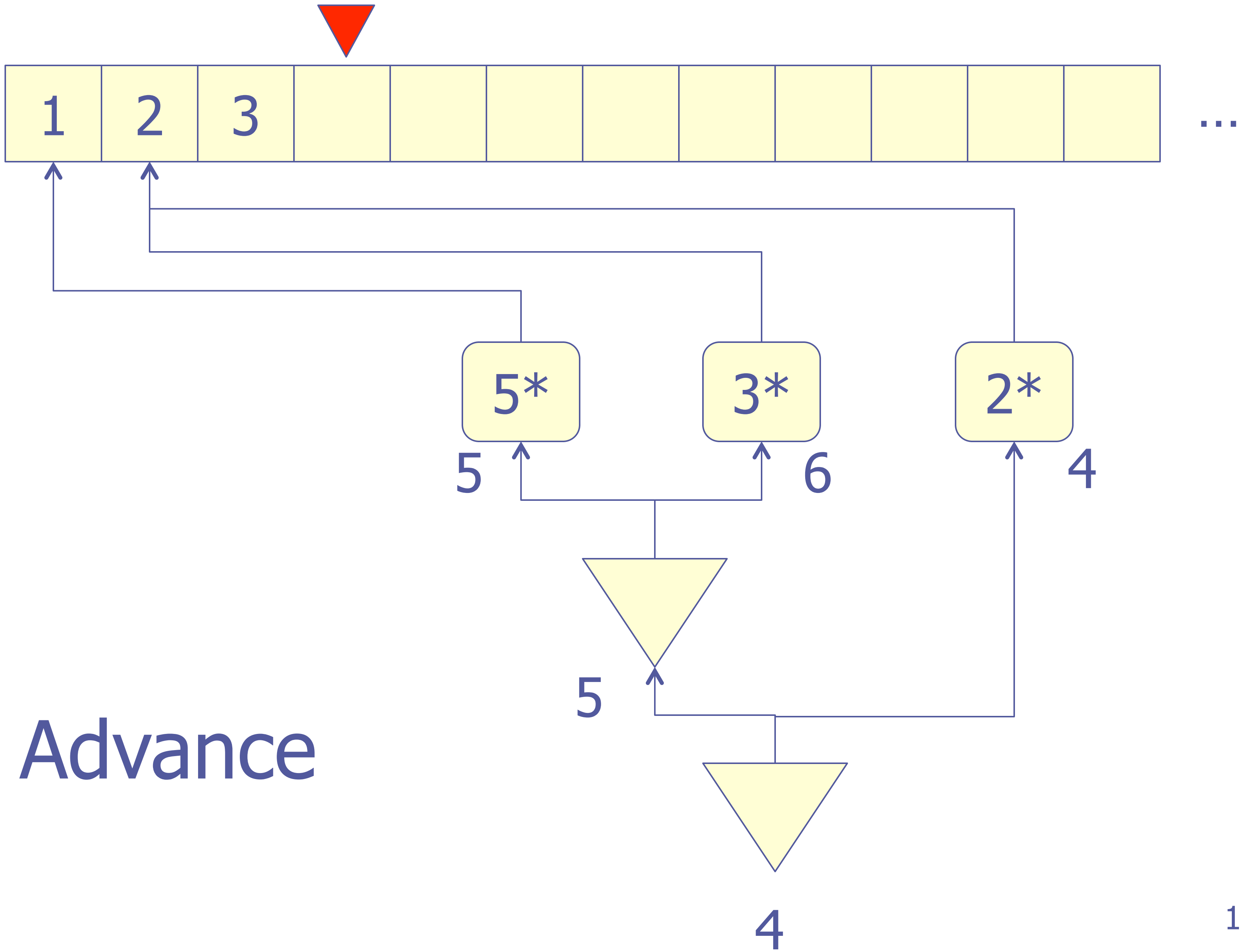
Advance

The Hamming Sequence:



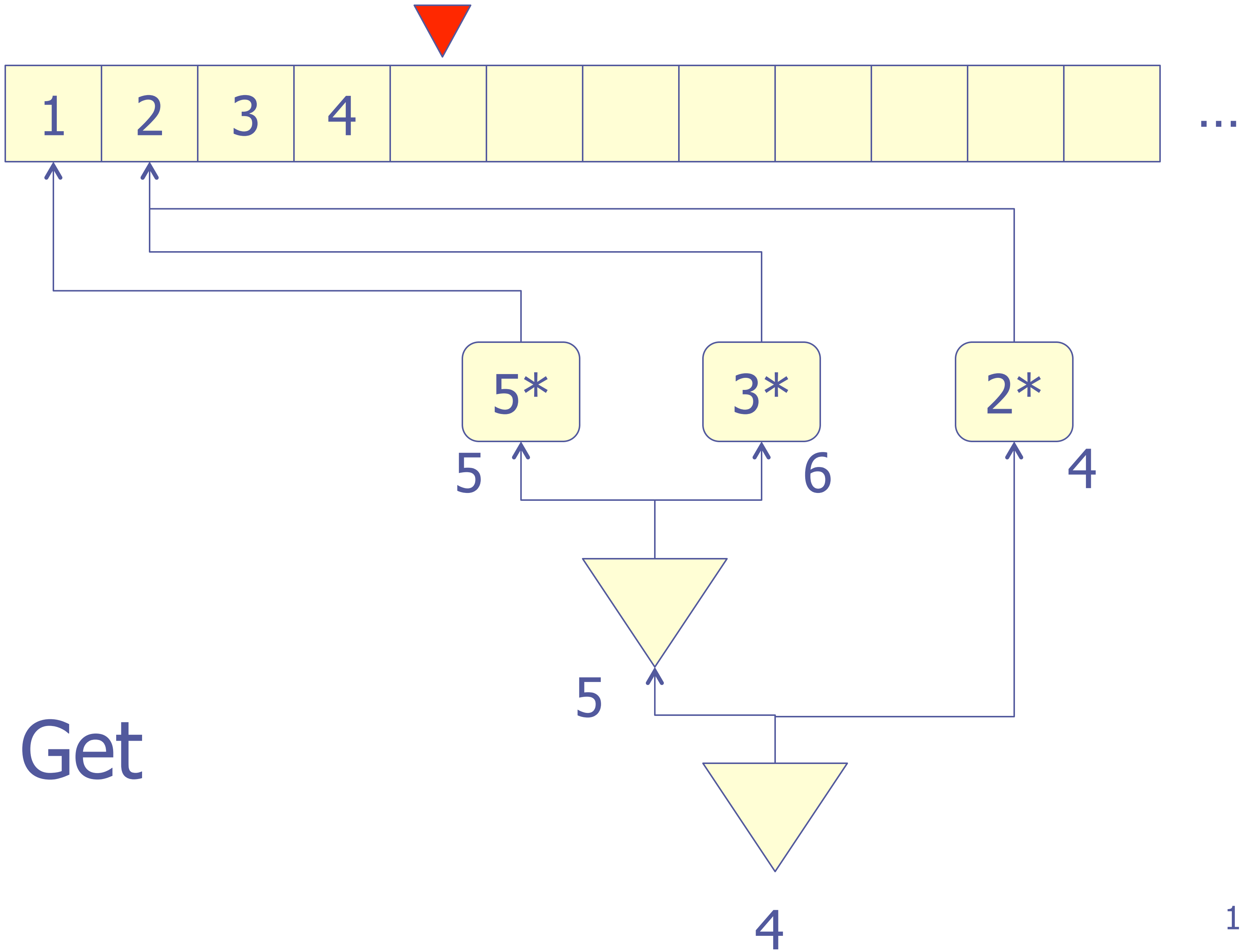
Get

The Hamming Sequence:



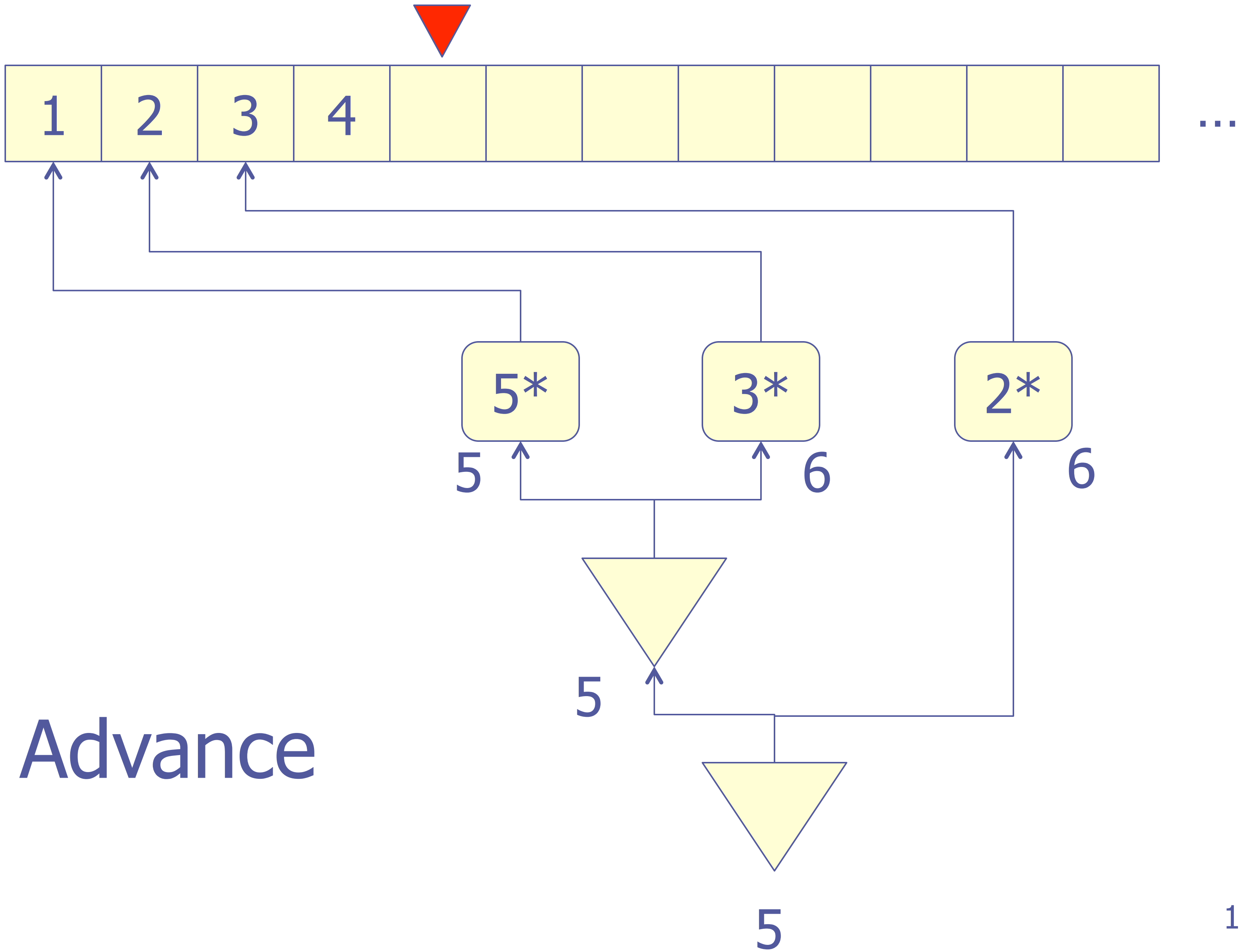
Advance

The Hamming Sequence:



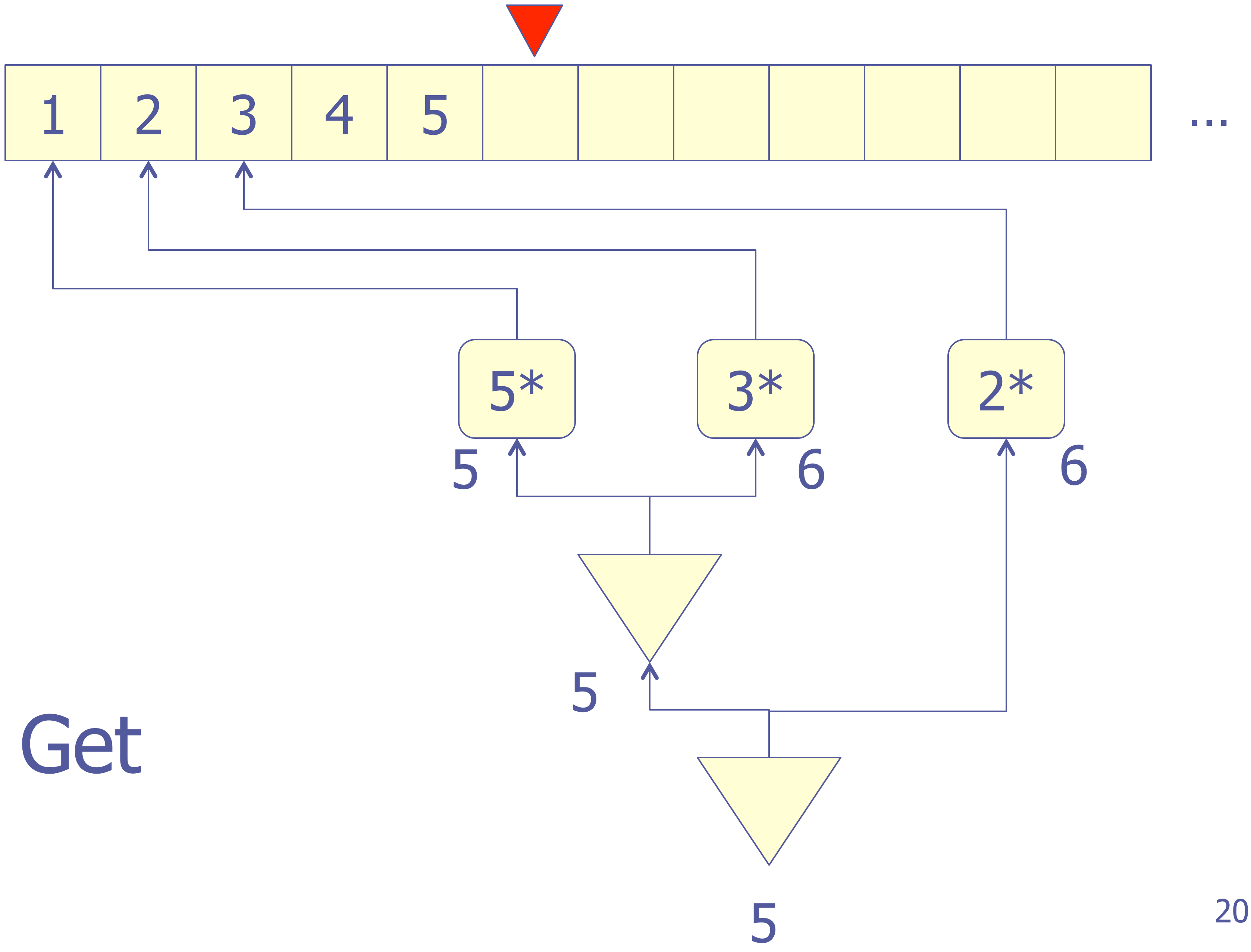
Get

The Hamming Sequence:



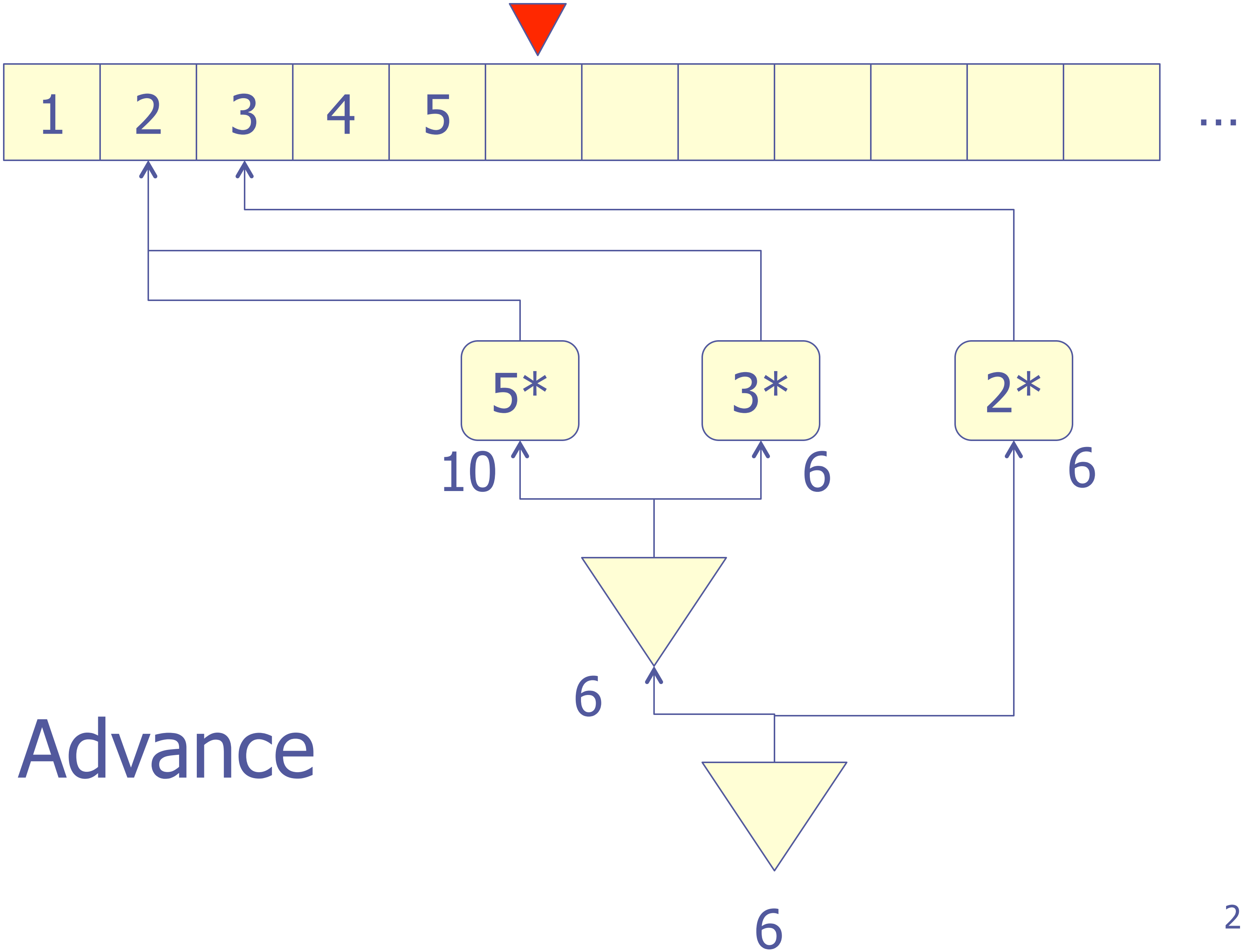
Advance

The Hamming Sequence:



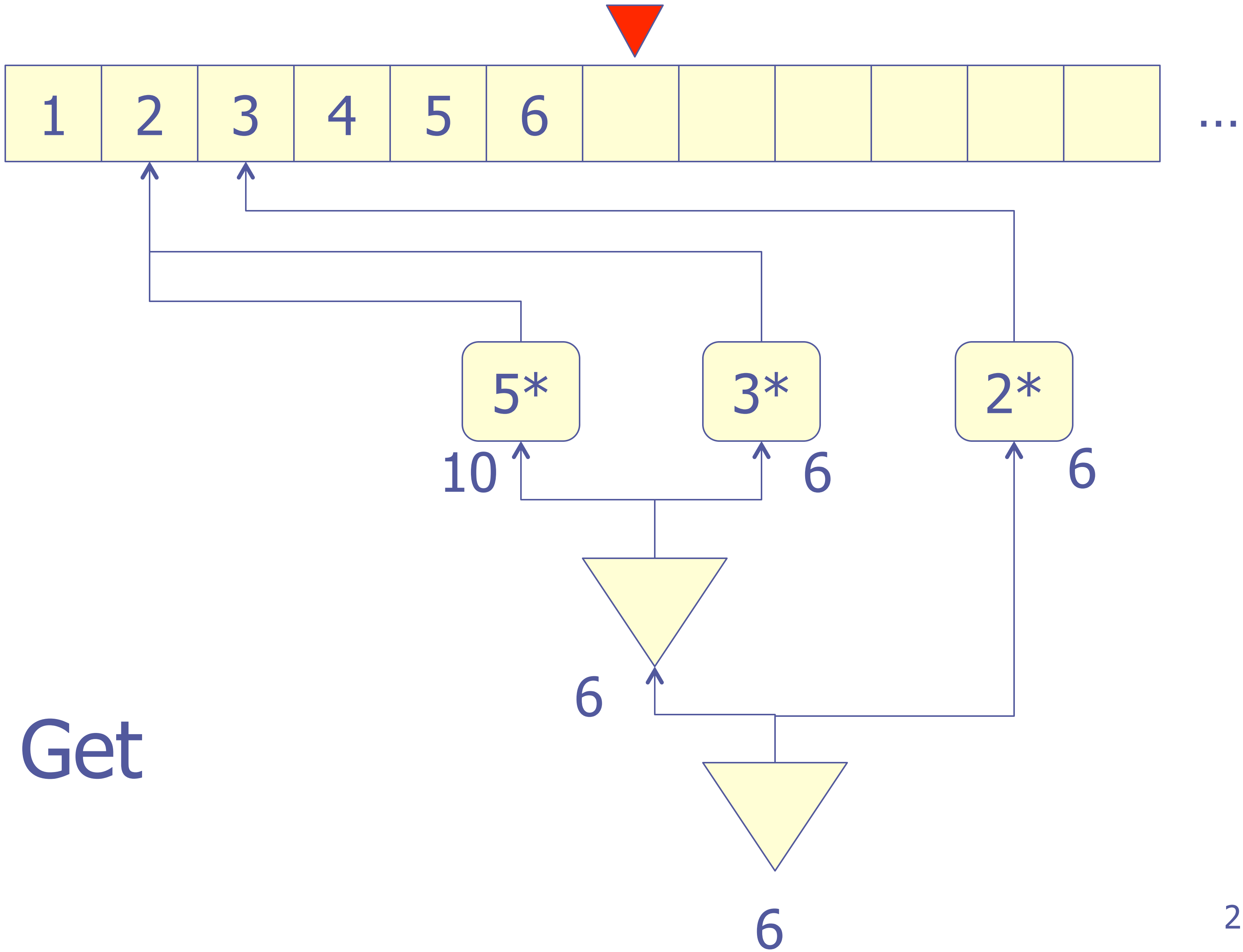
Get

The Hamming Sequence:



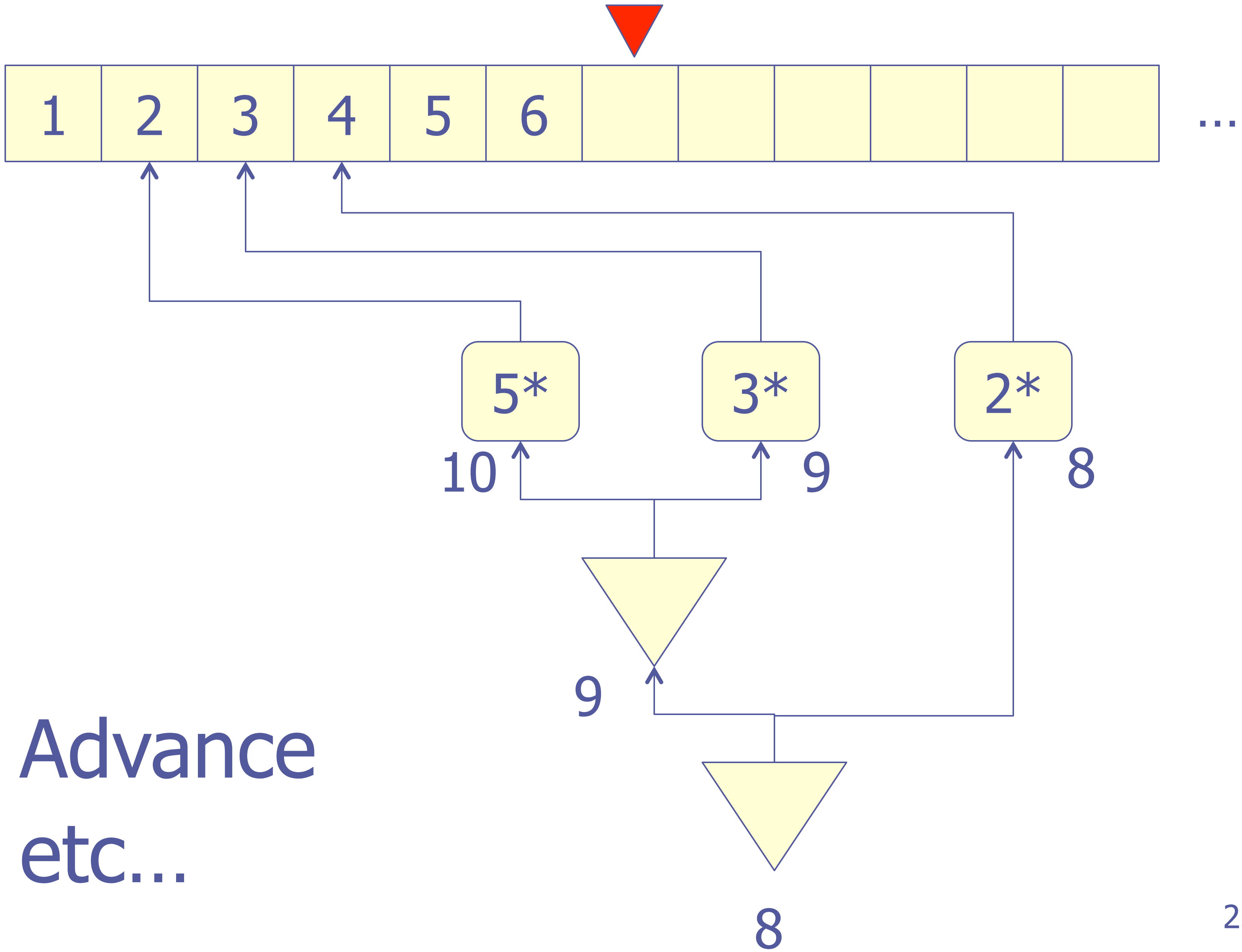
Advance

The Hamming Sequence:



Get

The Hamming Sequence:



Advance
etc…

Lists and Streams:

```
class List {
   int  head;
   List tail;
   List(int head) {
     this.head = head;
     this.tail = null;
   }
}
```

```
interface Stream {
  int  get();
  void advance();
}
```

Multiplier Streams:

```
class MultStream implements Stream {
  private int  mult;
  private List elems;
  MultStream(int mult, List elems) {
    this.mult  = mult;
    this.elems = elems;
  }

  public int get()           { return mult * elems.head; }
  public void advance() { elems = elems.tail; }
}
```

Merge Streams:

```
class MergeStream implements Stream {
  private Stream left, right;
  MergeStream(Stream left, Stream right) {
    this.left   = left;
    this.right = right;
  }

  public int get() {
    int l = left.get();
    int r = right.get();
    return (l<=r) ? l : r;
  }
```

Merge Streams (advance):

```
public void advance() {
  int l = left.get();
  int r = right.get();
  if (l == r) {
    left.advance();
    right.advance();
  } else if (l < r) {
    left.advance();
  } else {
    right.advance();
  }
```

Main Loop:

```
class Hamming {
  public static void main(String[] args) {
    List ham = new List(1);
    Stream s = new MergeStream(new MultStream(2, ham),
                 new MergeStream(new MultStream(3, ham),
                                 new MultStream(5, ham)));
    for (;;) {
      System.out.print(ham.head + ", ");
      int next = s.get();
      ham = ham.tail = new List(next);
      s.advance();
    }
  }
}
```

Observations:

- Hamming produces elements faster than the multiply/merge streams consume them
- We will never attempt to read uninitialized values
- The blue pointers are always behind the red pointer
- But the distance between the pointers will grow arbitrarily large … this can be considered a space leak

YAHS: (yet another Hamming solution)

```
factorOut          :: Int -> Int
factorOut n m
      | r == 0      = factorOut n q
      | otherwise = m
        where (q, r) = divMod m n

inHamming     :: Int -> Bool
inHamming      = (1==)
                 . factorOut 2
                 . factorOut 3
                 . factorOut 5
```

Summary:

- Programming with closures feels very natural in Haskell
  - Built-in support for lazy evaluation
  - Closure = function + arguments
  - Recursion

- But we can program with closures in other languages too!
  - One view of objects is as generalized closures:

    Instance variables = Data

    Methods = Multiple, parameterized Code entry points

- A powerful programming technique (not just for infinite lists)!

concat:

- concat :: [[a]] -> [a]
- concat [[1,2], [3,4,5], [6]]
  = [1,2,3,4,5,6]

- Laws:
  - filter p . concat = concat . map (filter p)
  - map f . concat = concat . map (map f)
  - concat . concat = concat . map concat

List Comprehensions:

General form:

- [expression | qualifiers]

where qualifiers are either:

- Generators: pat <- expr; or
- Guards: expr; or
- Local definitions: let defns

Works like a kind of generalized "for loop"

Examples:

```
[ x*x | x <- [1..6] ]
= [ 1, 4, 9, 16, 25, 36 ]

[ x | x <- [1..27], 28 `mod` x == 0 ]
= [ 1, 2, 4, 7, 14 ]

[ m | n <- [1..5], m <-[1..n] ]
= [ 1, 1,2, 1,2,3, 1,2,3,4, 1,2,3,4,5 ]
```

Applications:

- Some "old friends":

  ```
  map f xs        = [ f x | x <- xs ]
  filter p xs     = [ x | x <- xs, p x ]
  concat xss      = [ x | xs <- xss, x <- xs]
  ```

- Can you define take, head, or (++) using a comprehension?

Laws of Comprehensions:

```
[ x | x <- xs ]      = xs
[ e | x <- xs ]      = map (\x -> e) xs

[ e | True ]         = [ e ]
[ e | False ]        = []
```

$[\ e \mid gs_1, gs_2\ ] \quad = \text{concat}\ [\ [\ e \mid gs_2] \mid gs_1\ ]$

Example:

```
[ (x,y) | x <- [1,2], y <- [1,2] ]

= concat
      [ [ (x,y) | y <- [1,2]] | x <- [1,2] ]

= concat
      [ map (\y -> (x,y)) [1,2] | x <- [1,2] ]

= concat
      (map (\x ->
          map (\y -> (x,y)) [1,2]) [1,2])
```